Multirate Time-Domain Basis Functions for the Simulation of Pulse-Width-Modulation Controlled Devices

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***Abstract*—In this digest a new multirate approach for the time-stepping of electrical device models controlled by pulse-width modulation (PWM) is presented. The different time scales are seperated and periodic basis functions are introduced for the high dynamics such that a coarse time discretization can be performed on the latent scale. Eventually, this leads to a numerical scheme that can be very efficient for high PWM switching frequencies since the time steps are independent of the PWM.**

***Index Terms*—Circuit analysis, Finite element analysis, Harmonic analysis, Pulse width modulation**

I. Introduction to Multirate

Multiscale and multirate problems occur naturally in many applications from electrical engineering. Classical time discretization schemes are often inefficient. In particular for problems where some solution components are active while the majority is latent (e.g. behind a low-pass filter [1]) or problems with oscillatory solutions that are composed of multiple frequencies. For instance, a slowly varying signal is modulated onto a high frequency carrier, [2]. The latter scenario is well-known in the domain of radio frequencies but becomes more and more relevant also for low-frequency problems, e.g., control of electrical drives, because the switching frequencies of IGBTs increase while the power line frequency remains at 50-60Hz. Fig. 1 depicts some multitone solution components of an example problem (Buck Converter) at a modest switching frequency of 5000 Hz.

II. Numerical Method and Example

To this end, it is proposed to use a Galerkin approach in the time domain similar to harmonic balance [3], [4]. The j-th component of the solution is approximated by a sum of N_p terms

$$x_j = \sum_{k=0}^{N_p} p_k(t) x_{j,k}$$

where $p_k(t)$ are basis functions and $x_{j,k}$ coefficients. Often trigonometric functions are chosen as a basis but this can be generalized, e.g. using wavelets. For particular relevant engineering examples tailored basis functions (piecewise polynomial) can be constructed using analytical knowledge, as for the present case of a buck converter with PWM supply [5].

For continuous-conduction mode of the converter the first three basis functions are shown in Fig. 1. They are obtained recursively by integration, e.g.,

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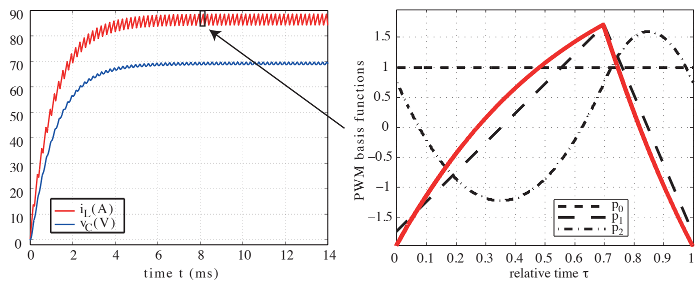

Fig. 1. Solution components of a buck converter operating with a PWM switching frequency of 5000 Hz with dedicated basis-functions.

$$p_1(\tau) = \begin{cases} \sqrt{3}\,\frac{2\tau - D}{D} & \text{if } 0 \le \tau \le D \\ \sqrt{3}\,\frac{1+D-2\tau}{1-D} & \text{if } D \le \tau \le 1 \end{cases},$$

where D is the duty cycle ($0 \le D \le 1$). The steady-state waveforms of the inductor current and the output voltage are obtained through the resolution of a system of algebraic equations. As the set of the PWM basis functions is enlarged (with increasing polynomial degree), the respective PWM ripple components are observed to converge very fast, e.g., $N_f = 4$ has been found sufficiently accurate for the present example (rel. err. in the order of 10^{-5}).

III. Conclusion

The idea of multirate/multitone integration has been illustrated using a simple buck converter example. In the full paper the extension for slowly varying coefficients $x_{j,k} = x_{j,k}(t)$ and the coupling to spatially resolved Finite Element models will be discussed.

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